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THE QLQG/LTR CONTROL FOR NONLINEAR SYSTEMS WITH A NON-GAUSSIAN NATURE

Jong Shik Kim*

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The QLQG/LTR control method is applied to a nonlinear system with Coulomb friction which has a non-Gaussian nature. It is shown that the non-Gaussian nature degrades the effectiveness of the QLQG/LTR design method. Thus, a method for alleviating this problem is proposed. It is the QLQG/LTR control method with a modified model based compensator which can consider the non-Gaussian nature of nonlinear systems. The computer simulation results show that the responses of this nonlinear control system are relatively insensitive to the input magnitude even if there exist a hard nonlinearity and a non-Gaussian nature in the plant.

Key Words : Statistical Linearization, LQG/LTR, QLQG/LTR, Model Based Compensator

1. INTRODUCTION

The QLQG/LTR(Quasi-Linear Quadratic Gaussian with Loop Transfer Recovery) control method is a powerful one for designing controllers of nonlinear systems with hard nonlinearities such as Coulomb friction backlash and saturation. This method is the integration of statistical linearization (Gelb and Vander Velde, 1968) and loop shaping techniques (Doyle and Stein, 1981).

Statistical linearization techniques included in the QLQG/LTR method are usually used under the Gaussian assumption. But, sometimes the accuracy of statistical linearization is important. It depends on the amount of distortion produced by the nonlinearity and the effectiveness of the low pass part of the plant. And, it can be improved by using non-Gaussian statistics (Assaf 1976; Beaman, 1979). However, non-Gaussian statistics are not suitable for applying to QLQG/LTR control systems. Because, it is very difficult to compute the random input describing function gains for nonlinearities by non-Gaussian statistics. Thus, in this paper, a modified QLQG/LTR method is proposed which can improve the performance and stability-robustness of nonlinear systems with a strong non-Gaussian nature.

As a design example, a simple first order nonlinear system with Coulomb firction is selected. And, the linear controller using the LQG/LTR method and the nonlinear controller using the QLQG/LTR method with a model based compensator and with a modified model based compensator are synthesized and compared.

2. QLQG/LTR CONTROL METHOD

Nonlinear plant dynamics can be expressed as follows :

$$\dot{x}(t)=f(x(t))+Bu(t)+\Gamma w(t) \tag{1}$$

*Department of Mechanical Design Engineering, Pusan National University, Pusan 609-735, Korea,

where

$x(t)$ is the $(n\times 1)$ plant state vector,
$f(x(t))$ is an $(n\times 1)$ vector,
$u(t)$ is the $(m\times 1)$ control input vector,
$w(t)$ is the $(p\times 1)$ disturbance input vector.

We assume that all the nonlinearities are symmetric and single-valued. Then, the nonlinear plant dynamics(1) can be linearized via statistical linearization techniques.

- Statistically linearized plant

$$\dot{x}(t)=N(\sigma_x)x(t)+Bu(t)+\Gamma w(t) \tag{2}$$

where
$N(\sigma_x)$ is the $(n\times n)$ statistically linearized plant matrix,
σ_x is the standard deviation of the plant states.

- Measurement

$$y(t)=Cx(t)+v(t) \tag{3}$$

where

$y(t)$ is the $(m\times 1)$ measured output vector,
$v(t)$ is the $(m\times 1)$ measurement noise vector.

- Control

$$u(t)=-Gz(t) \tag{4}$$

where

G is the $(m\times n)$ control gain matrix,
$z(t)$ is the $(n\times 1)$ compensator state vector.

- MBC(Model Based Compensator)

$$\dot{z}(t)=N(\sigma_z)z(t)+Bu(t)+H(y(t)-Cz(t)-r(t)) \tag{5}$$

where

$N(\sigma_z)$ is the $(n\times n)$ statistically linearized compensator matrix,
σ_z is the standard deviation of the compensator states,
H is the $(n\times m)$ filter gain matrix,
$r(t)$ is the $(m\times 1)$ command input vector.

By combining Eqs. (2) and (5), the statistically linearized compensated plant dynamics can be expressed as follows:

$$\begin{Bmatrix}\dot{x}(t)\\ \dot{z}(t)\end{Bmatrix}=\begin{bmatrix}N(\sigma_x) & -BG\\ HC & N(\sigma_z)-HC-BG\end{bmatrix}\begin{Bmatrix}x(t)\\ z(t)\end{Bmatrix}+\begin{bmatrix}0 & \Gamma & 0\\ -H & 0 & H\end{bmatrix}\begin{Bmatrix}r(t)\\ w(t)\\ v(t)\end{Bmatrix} \tag{6}$$

In Eq. (6), the statistically linearized values of the elements of the plant ($N(\sigma_x)$) and compensator ($N(\sigma_z)$) are the same since the compensator is a model of the statistically linearized plant. However, since the statistics of the plant and compensator states are different it follows that the nonlinear functions which produce these statistically linearized elements, in general, will be different.

Now, for selecting desired design matrices(G and H) systematically, let us use the separation property (Kwakernaak and Sivan, 1972) which is one of the special properties of the MBC. We can choose the state vector for closed-loop system $x_c(t)\in R^{2n}$ in order to describe the separation property.

$$x_c(t)=\begin{Bmatrix}x(t)\\ \tilde{x}(t)\end{Bmatrix}$$

where

$$\tilde{x}(t)=x(t)-z(t) \tag{7}$$

Then the statistically linearized QLQG/LTR control system can be expressed as follows:

$$\dot{x}_c(t)=\begin{bmatrix}N-BG & -BG\\ 0 & N-HC\end{bmatrix}x_c(t)+\begin{bmatrix}0 & \Gamma & 0\\ H & \Gamma & -H\end{bmatrix}\begin{Bmatrix}r(t)\\ w(t)\\ v(t)\end{Bmatrix} \tag{8}$$
$$y(t)=[C\ \ 0]x_c(t)$$

The $2n$ statistically linearized eigenvalues can be separated into two distinct groups [$\det(\lambda I-N+BG)$ and $\det(\lambda I-N+HC)$]. It is now clear that the compensator design decomposes into finding G and H. We can select H from the TFL(Target Filter Loop) design and G from the LTR(Loop Transfer Recovery) procedure separately.

The filter gain matrix H is calculated by

$$H=\frac{1}{\mu}PC^T \tag{9}$$

where P is the solution of the FARE(Filter Algebraic Riccati Equation):

$$NP+PN^T+LL^T-\frac{1}{\mu}PC^TCP=0 \tag{10}$$

Matrix L and scalar μ in the FARE are used as design parameters for the desired loop-shaping(Athans, 1986). And, the control gain matrix G is calculated by

$$G=\frac{1}{\rho}B^TS \tag{11}$$

where S is the solution of the CARE(Control Algebraic Riccati Equation):

$$SN+N^TS+C^TC-\frac{1}{\rho}SBB^TS=0 \tag{12}$$

Scalar ρ in the CARE is used as a design parameter for the LTR(Athans, 1986).

Finally we must be careful to calculate the stationary statistics of the system. Since the driving noises of real system and the fictitious noises are in general different for the design purpose, the Lyapunov equation for the compensated plant should be solved for the calculation of the DF(Describing Function) gains and stationary statistics of the system. The Lyapunov Equation for the compensated plant can be derived from Eq. (6).

$$N_tX_t+X_tN_t^T+\Gamma_tW_t\Gamma_t^T=0 \tag{13}$$

where

$$N_t=\begin{bmatrix}N & -BG\\ HC & N-BG-HC\end{bmatrix},\ X_t=\begin{bmatrix}X & Y\\ Y & Z\end{bmatrix},$$
$$\Gamma_t=\begin{bmatrix}0 & \Gamma & 0\\ -H & 0 & H\end{bmatrix},\ W_t=\begin{bmatrix}R & 0 & 0\\ 0 & W & 0\\ 0 & 0 & v\end{bmatrix},$$
$$X=E[x(t)x(t)^T],\ Y=E[z(t)\tilde{x}(t)^T],$$
$$Z=E[z(t)z(t)^T],\ R=E[r(t)r(t)^T],$$
$$W=E[w(t)w(t)^T],\ V=E[v(t)v(t)^T].$$

The design procedure of the QLQG/LTR system is as follows:

(1) Determine a mathematical model for the nonlinear plant to be controlled.

(2) Analyze the linearized system via statistical linearization techniques.

(3) Determine the design specification.

(4) Determine the several zero mean white noise inputs which should be represented an operating range of interest.

(5) Select an operating point to design a linear controller.

(6) Estimate the DF gains for the nonlinearities at the selected operating point.

(7) Do loop shaping of the TFL.

(8) LTR using the cheap control quasi-linear quadratic regulator problem.

(9) Slove the Lyapunov equation for the compensated plant.

(10) Calculate the DF gains for nonlinearities.

(11) Compare the estimated DF gains with the computed ones and repeat steps (6) through (11) until the difference between them is small enough.

(12) Store the gains(filter, control and DF) and the standard deviations(compensator states and filter innovations).

(13) Repeat the design procedure from steps (5) through (12) for each operating point.

(14) Determine the relationships between the gains(filter, control and DF) and the stationary statistics of the systems. i, e., $H(\sigma_f)$, $G(\sigma_z)$ and $N(\sigma_z)$ where σ_f and σ_z are the

standard deviations of the filter innovations and compensator states, respectively.

(15) Synthesize the desired nonlinear functions via the inverse random input describing function techniques.

(16) Implement the final nonlinear controller and check the time responses of outputs and control inputs.

The above design procedure is about the QLQG/LTR control method with a MBC, which is presented at the Kim's paper(Kim, 1989) in detail. For nonlinear systems with a weak non-Gaussian nature, the QLQG/LTR control system with a MBC has no problem. However, for nonlinear systems with a strong non-Gaussian nature, the non-Gaussian nature of nonlinear systems and the accuracy of statistical linearization should be checked. And, a modified MBC is designed with these results to compensate the errors of the statistics by the Gaussian assumption. The concrete design method of the modified MBC will be discussed with the following design example.

3. DESIGN EXAMPLE

3.1 Problem Formulation

A simple first order nonlinear system with Coulomb friction is selected as a design example to illustrate the QLQG/LTR control method with a modified MBC.

$$\begin{cases} \dot{x}_p(t) = -\mathrm{sgn}(x_p(t)) + u_p(t) \\ y(t) = x_p(t) \end{cases} \tag{14}$$

where $x_p(t)$ is the plant state, $y(t)$ is the output, and $u_p(t)$ is the control input. The above nonlinear plant can be linearized via statistical linearization techniques. Then, the statistically linearized plant is expressed as follows:

$$\begin{cases} \dot{x}_p(t) = -N_c(\sigma_x)x_p(t) + u_p(t) \\ y(t) = x_p(t) \end{cases} \tag{15}$$

where $N_c(\sigma_x)$ is the DF gain for Coulomb friction ($N_c(\sigma_x) = (\sqrt{2/\pi})/\sigma_x$) and σ_x is the standard deviation of $x_p(t)$.

And, the design specifications considered are as follows:

(1) Steady state tracking error should be zero for an arbitrary constant input.

(2) Gain crossover frequency should be about 10 rad/sec.

(3) The singular value of the sensitivity TF(Transfer Function) should be less than -20db for all $\omega<1$ rad/sec for the good command following and disturbance rejection.

(4) The singular value of the closed-loop TF should be less than -20db for all $\omega>100$rad/sec for the stability-robustness to unmodelled dynamics and insensitivity to sensor noise.

To meet the design specification(1), it is necessary to augment an integrator at the plant input. Then, the DPM(Design Plant Model) is expressed as follows:

$$\begin{cases} \dot{x}(t) = N(\sigma_x)x(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{16}$$

where

$$x(t) = \begin{Bmatrix} u_p \\ x_p \end{Bmatrix},\ N(\sigma_x) = \begin{bmatrix} 0 & 0 \\ 1 & -N_c(\sigma_x) \end{bmatrix},$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } C = [0\ 1]$$

3.2 Linear Controller Design Using the LQG/LTR Method

We should have a linear plant in order to use the LQG/LTR(Linear Quadratic Gaussian with Loop Transfer Recovery) method. The Coulomb friction nonlinearity ($\mathrm{sgn}(x_p)$) is assumed as a linear one (x_p), which can represent a linear gain for a large operating range inclusive of small and large operating conditions. Then, the DPM dynamics are expressed as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu \\ y(t) = Cx(t) \end{cases} \tag{17}$$

where

$$A = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } C = [0\ 1]$$

The DPM is found to be completely controllable from the input $u(t)$ and completely observable through the output $y(t)$, and is also a minimum phase plant. Therefore, we can design the LQG/LTR compensator with a guarantee of the LTR.

The TFL design which is the first procedure of the LQG/LTR method can be accomplished by cancelling the open-loop pole, leaving only the augmented integrator, which provides an 'optimal' loop shape (Kim, 1988). Then, the design parameter L is calculated by

$$L = Z_c^{-1} z_d = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{18}$$

where Z_c is the (2×2) matrix containing as its columns the coefficients of the constituent zero polynomials of the $G_{FOL}(s)(=C(sI-A)^{-1}L)$ transfer function, and z_d is the desired zero polynomial.

To determine the filter gain matrix H, the desired crossover frequency was specified as 10 rad/sec. A value of 0.01 for μ is found to provide a crossover frequency of 10 rad/sec for the TFL which is shown in Fig. 1.

After selecting L and μ to satisfy the desired target filter loop shaping, we calculate the filter gain matrix H from Eqs. (9) and (10). The resulting filter gain matrix H is:

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix} \tag{19}$$

The LTR which is the second procedure of the LQG/LTR method is attempted with the cheap control linear quadratic regulator problem. We usually recover the TFL up to a decade beyond crossover frequency. This level of recovery is

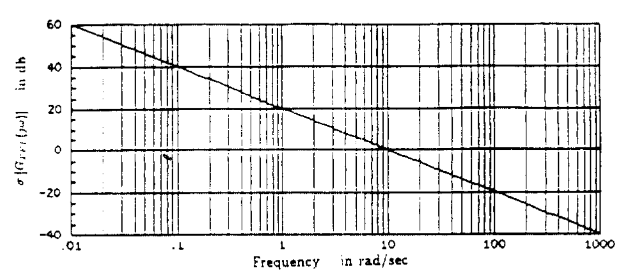

Fig. 1 Singular value of the target filter loop TF

obtained with $\rho=10^{-9}$. Then the control gain matrix G is calculate from Eqs. (11) and (12). The resulting control gain matrix G is:

$$G=[G_1\ G_2]=[250\ \ 31372] \tag{20}$$

The singular value of the recovered loop TF for the assumed linear plant with the LQG/LTR compensator is shown in Fig. 2, which is satisfactory to meet the design specfications for the assumed linear plant.

Now let us check the performance and stability-robustness for the nonlinear plant with the LQG/LTR compensator. For this purpose, we check the frequency responses for 3 different command inputs, which are assumed as zero mean white noises for the statistical linearization of the nonlinear plant. The white noise intensities of the selected command inputs (R) are 100, 0.1 and 0.001 which represent large, medium and small input cases, respectively. Then, the singular value plot of the loop TF and the normalized step responses for the nonlinear plant with LQG/LTR compensator are shown in Fig. 3 and Fig. 4.

It is found that the LQG/LTR control system satisfies the stability-robustness condition for any input, but it does not satisfy the performance requirements for small inputs. The LQG/LTR compensator can be used with good system performance for the input 1 or so. When the input is larger than 1, there is about 7% overshoot and when the input is smaller than 1, the settling time increases up to 6 times of the specified one. Therefore, we cannot obtain the desirable response with a linear LQG/LTR compensator for a large operating range. It is because that the transfer function of the linear LQG/LTR compensator can not vary according to the input magnitude. In other words, since the system parameters of the nonlinear plant depend on the input magnitude, a nonlinear compensator which can adapt to the changes in input is required.

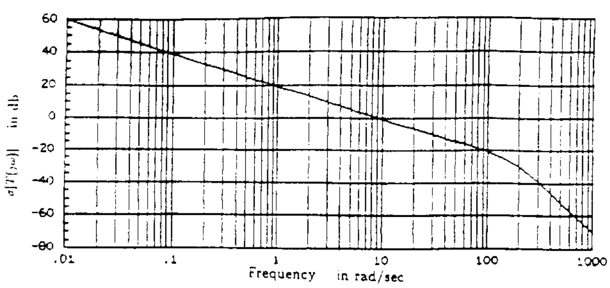

Fig. 2 Singular value of the loop TF for the linear plant with the LQG/LTR compensator

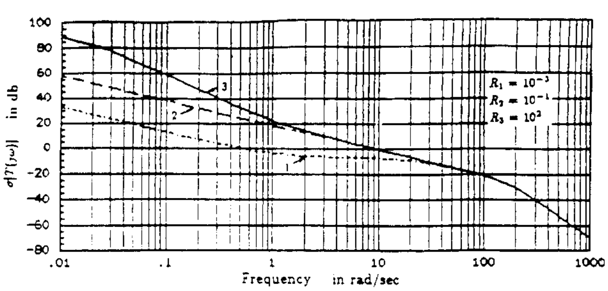

Fig. 3 Singular value of the loop TF for the nonlinear plant with the LQG/LTR compensator

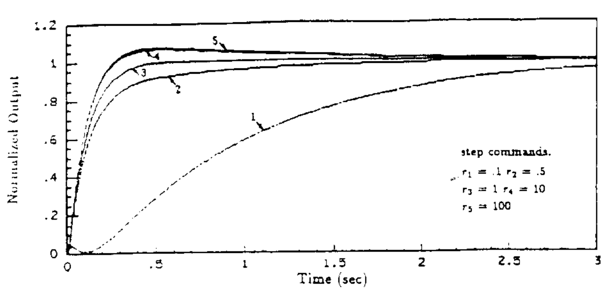

Fig. 4 Step responses for the LQG/LTR control system

3.3 Nonlinear Controller Design Using the QLQG/LTR Method with a MBC

We should have the statistically linearized plant and select several operating points to cover an operating range of interest to apply the QLQG/LTR method. Thus command inputs are assumed as zero mean white noises of which intensities (R) are between 10^{-5} and 100. The QLQG/LTR design procedure is executed for a linear design at each selected operating point. Although the final results of all the designs are provided, only one design procedure will be discussed here. The medium input case ($R=0.1$) is chosen for this purpose.

The DF gain for Coulomb friction is computed as 1.15 when R is 0.1, and the TFL is the same as the LQG/LTR case which is shown in Fig. 1. The design parameter L is calculated by Eq. (18) and μ is selected as 0.01 to satisfy the crossover frequency. After selecting L and μ to satisfy the desired target filter loop shaping, we calculate the filter gain matrix H from Eqs. (9) and (10). The resulting filter gain matrix H is the same as the LQG/LTR case.

The LTR is attempted with the cheap contol quasi-linear quadratic regulator problem (Kim, 1989) in which the value of 10^{-9} for the control weighting parameter ρ is used. Then the control gain matrix G is calculated from Eqs. (11) and (12). The resulting control gain matrix G is:

$$G=[G_1\ G_2]=[251\ \ 31478] \tag{21}$$

In the similar manner, we can apply the QLQG/LTR design procedure for the other command inputs. Then, the singular value plot of the loop TF for the QLQG/LTR control system is shown in Fig. 5.

The gains(filter, control and DF) and the stationary statistics (compensator states and filter innovation) are stored for

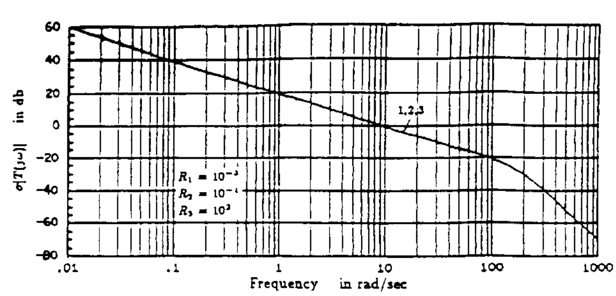

Fig. 5 Singular value of the loop TF for the QLQG/LTR control system.

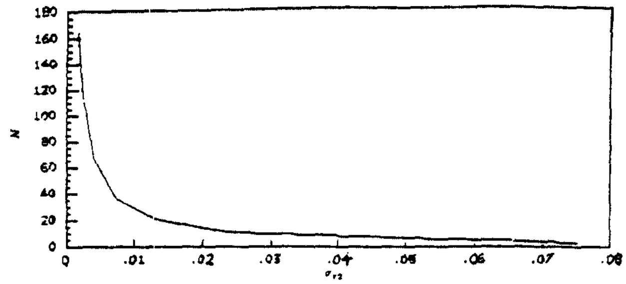

Fig. 6(a) Quasi-linear gain N versus σz_2

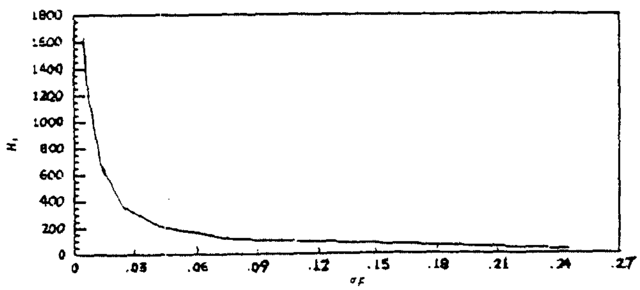

Fig. 6(b) Quasi-linear gain H_1 varsus σ_f

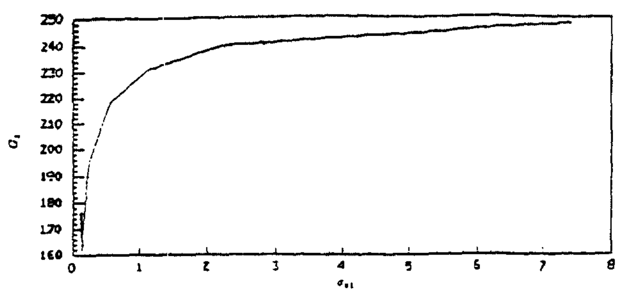

Fig. 6(c) Quasi-linear gain G_1 versus σ_{z1}

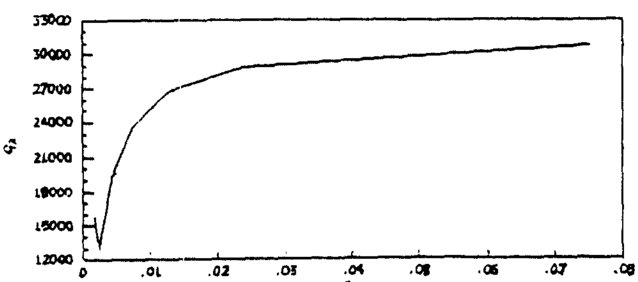

Fig. 6(d) Quasi-linear gain G_2 versus σ_{z2}

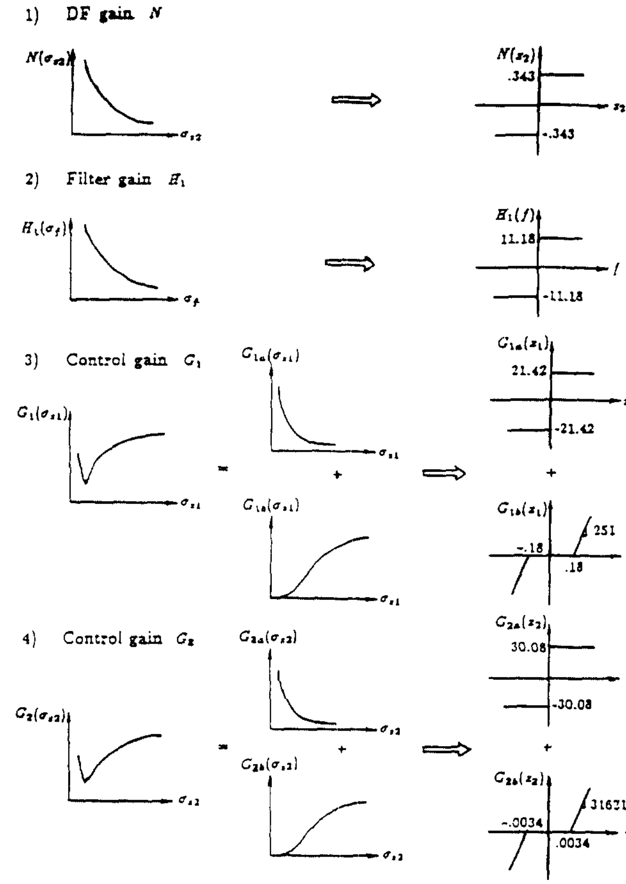

Fig. 7 Desired nonlinear functions via the inverse random input describing function techniques

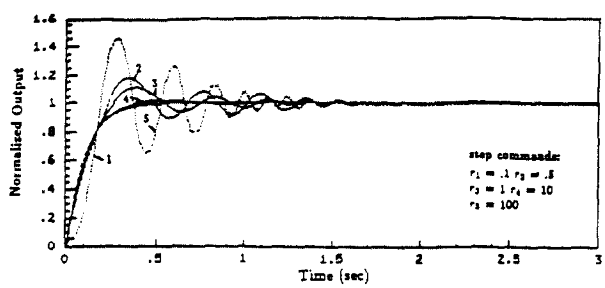

Fig. 8 Step responses for the QLQG/LTR control system

all linear designs. The relationships between the gains and the stationary statistics, i.e., $N(\sigma_{z2})$, $H_1(\sigma_f)$, $G_1(\sigma_{z1})$ and $G_2(\sigma_{z2})$ are shown in Fig. 6(a) through Fig. 6(d) and H_2 is 10 for any input.

The nonlinear gain functions are synthesized from the set of statistically linearized gains obtained from the linear design results. This procedure can be executed via the inverse random input describing function techniques(Suzuki and Hedrick, 1985). The final results of the nonlinear functions are shown in Fig. 7. After obtaining the desired nonlinear functions via the inverse random input describing function techniques, we can synthesize the final nonlinear feedback control system. The step responses for the nonlinear plant with the nonlinear QLQG/LTR compensator are shown in Fig. 8.

The QLQG/LTR control system with a MBC satisfies the design specifications in the frequency domain for all the input range, but the time responses are not satisfactory. For large inputs, the settling time is about 0.5 seconds and no overshoot exists. However, as the input magnitude decreases, the system performances become bad. For example, when the input magnitude is 0.1, there exists about 50% overshoot and chattering phenomena are observed. These undesirable responses probably result from the worng estimation of the

DF gain due to the non-Gaussian nature of the nonlinear plant. Therefore, if there are undesirable responses in the QLQG/LTR control system with a MBC, we should check the accuracy of statistical linearization. It will be discussed in th following section.

3.4 Nonlinear Controller Design Using the QLQG/LTR Method with a Modified MBC

In the system responses of the QLQG/LTR control system for small inputs, there exist large overshoot and chattering phenomena, These bad responses are due to the accuracy of statistical linearization. We can find the non-Gaussian nature of the QLQG/LTR control system for small inputs from Fig. 9. Therefore, there are some deviations between the values estimated by the Gaussian statistical linearization and Monte Carlo simulation results for small input case, which are shown in Fig. 10.

From Fig. 10, it is found that the real DF gains for Coulomb friction are small compared to the estimated DF gains under the assumption of a Gaussian process for small inputs. The relative errors of the estimated standard deviations of the input to the Coulomb friction are given in Table 1.

In Table 1, σ_x^E and σ_x^M are the standard deviations of the input to the Coulomb friction by estimation and by Monte Carlo simulation, respectively. And, "Error" means $(\sigma_x^M - \sigma_x^E)/\sigma_x^M \times 100(\%)$.

In order to alleviate the deviations by the non-Gaussian nature, a modified MBC is proposed as follows. The Coulomb friction nonlinearity is modified as a saturation nonlinearity with an appropriate saturation point and the same maximum value. First, in order to find an appropriate saturation point, we should investigate the DF gains for Coulomb friction ($N_c = \sqrt{2/\pi}/\sigma_x$) and saturation ($N_s = (1/\delta)\mathrm{erf}(\delta/\sqrt{2}\sigma_x)$) where σ_x is the standard deviation of the input to the nonlinearity

Table 1 Errors of the estimated standard deviations

R	σ_x^E	σ_x^M	Error(%)
10^{-5}	.0069	.0113	38.85
10^{-4}	.0219	.0266	17.76
10^{-3}	.0693	.0725	4.41
10^{-2}	.2191	.2214	1.05
10^{-1}	.6930	.6960	.43
1	2.191	2.201	.45
10	6.930	6.960	.43
10^2	21.91	22.01	.44

and δ is a saturation point. The two DF gains are almost the same for the large value of σ_x/δ. The difference between the two gains is 1% when σ_x/δ is 3. From Table 1, the error of the standard deviation is approximately 1% when σ_x/δ is about 0.3. With this result, the Coulomb friction nonlinearity is modified as a saturation nonlinearity with saturation point of 0.1 to compensate the errors by the Gaussian assumption. Next, the QLQG/LTR control method is applied for this modified nonlinear plant. Then, the singular value plot of the loop TF and the normalized step responses for the QLQG/LTR control system with a modified MBC are shown in Fig. 11 and Fig. 12, respectively.

The results of the QLQG/LTR control system with a modified MBC are satisfactory for a large operating range. The maximum overshoot is about 17% and no chattering exists in the considered operating range. The settling time in about 0.5 seconds for large inputs and 0.8 seconds for small inputs. And, this nonlinear control system is relatively insensitive to the input magnitude.

In conclusion, the system performances can be improved tremendously by using a modified QLQG/LTR compensator for the nonlinear system with a non-Gaussian nature. The final nonlinear feedback control system using a modified

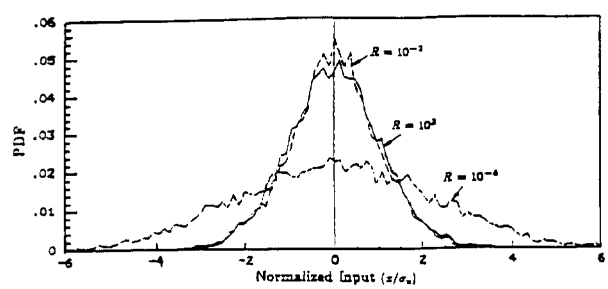

Fig. 9 Probability density function for the input to the Coulomb friction in the QLQG/LTR control system

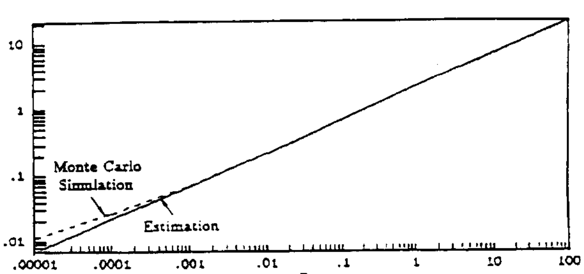

Fig. 10 Comparison of standard deviations by estimation and 150 run Monte Carlo simulation

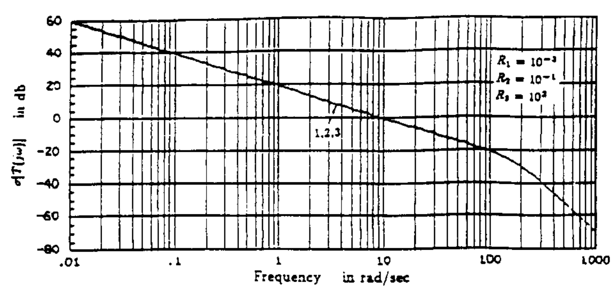

Fig. 11 Singular value of the loop TF for the QLQG/LTR control system with a modified MBC

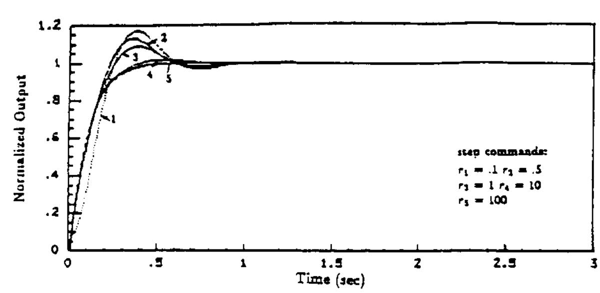

Fig. 12 Step responses for the QLQG/LTR control system with a modified MBC

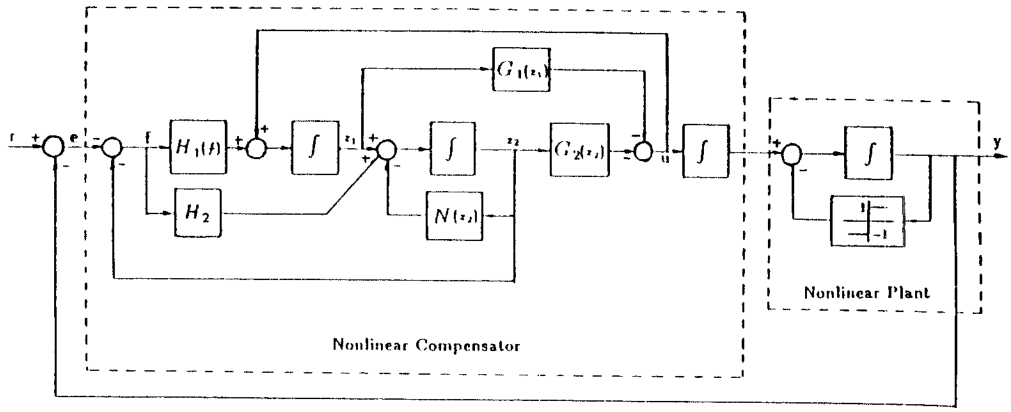

Fig. 13 Nonlinear QLQG/LTR control system with a modified MBC

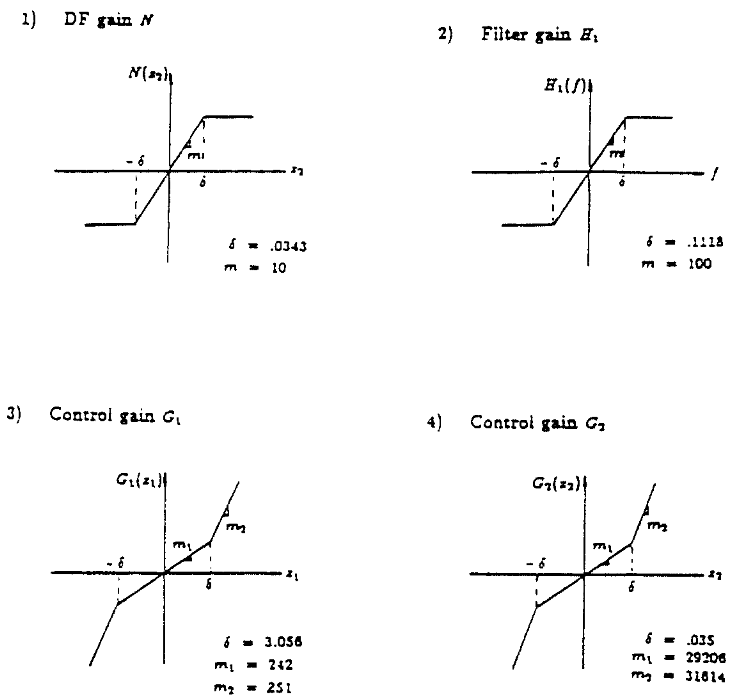

Fig. 14 Desired nonlinear functions for the modified QLQG/LTR compensator

QLQG/LTR compensator is shown in Fig. 13 and the desired nonlinear functions for the modifed QLQG/LTR compensator are shown in Fig. 14 in detail.

In order to check the accuracy of the statistical linearization for the QLQG/LTR control system with a modified MBC, the PDF(Probability Density Function) is obtained from the simulation result. The PDF for the input to the Coulomb friction nonlinearity in the modified QLQG/LTR control system is showin in Fig. 15. The PDF has almost a Gaussian shape in the considered operation range. And, the errors of standard deviation of the input to the Coulomb friction nonlinearity for each control system are shown in Fig. 16. It is found that the estimated DF gains are almost the same as the real ones for the QLQG/LTR control system with a modified MBC.

The degree of non-Gaussian nature depends on the structure of nonlinear systems, and the non-Gaussian nature

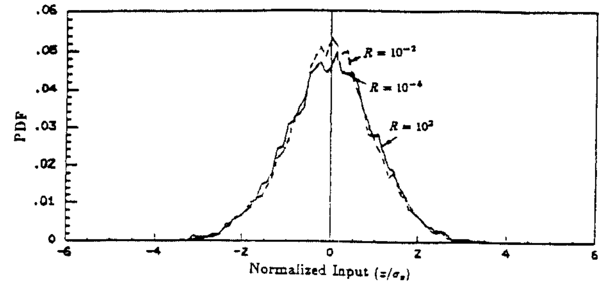

Fig. 15 Probability density function for the input to the Coulomb friction in the modified QLQG/LTR Control system

such as chattering phenomena and large overshoot. In order to get rid of them, a modified QLQG/LTR method has been developed. The plant model is modified to alleviate the errors

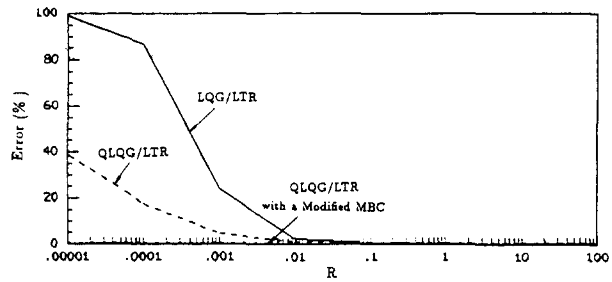

Fig. 16 Errors of the standard deviations of the input to the Coulomb friction

makes it difficult to estimate the DF gains for nonlinearities. Therefore, it is desirable that we first try to design the QLQG/LTR compensator with a MBC under the assumption of a Gaussian process. If an undesirable response (large overshoot or chattering) is apparent, the PDF and the standard deviation of the input to the nonlinearity should be checked. With these results, we may modify the nonlinearity to alleviate the deviations between the estimated and Monte Carlo simulation results and we synthesize a modified QLQG/LTR compensator based on this nonlinear plant.

4. SUMMARY AND CONC LUSIONS

The LQG/LTR, QLQG/LTR and modified QLQG/LTR compensators are synthesized for a nonlinear system with Coulomb friction which has a non-Gaussian nature. After synthesizing the compensators, the system responses are compared by computer simulation in both frequency and time domains.

It is found that the LQG/LTR compensator can be used only for a small operating range. This means that the LQG/LTR compensator is suitable only when the nonlinearity is not severe. And, it is also found that the QLQG/LTR compensator can be used for a relatively large operating range compared to the LQG/LTR compensator. However, the time responses of the QLQG/LTR control system are not satisfactory for all the operating range. This is due to the accuracy of statistical linearization.

In case that nonlinear systems have a strong non-Gaussian nature the DF gains cannot be estimated accurately under the Gaussian assumption. This results in undesirable responses of the statistics by the Gaussian assumption. To put it concretely, the Coulomb friction nonlinearity is modified as a saturation nonlinearity with a saturation point($\delta=0.1$) and the same maximum value. And, the QLQG/LTR control method with a MBC is applied for this modified nonlinear plant. Then the system performances of the nonlinear system with a non-Gaussian nature can be improved as compared with the QLQG/LTR control system for the original nonlinear plant. The responses of this nonlinear control system have small overshoot, no chattering and fast settling time at the large operating range even if there exist a hard nonlinearity and a non-Gaussian nature in the plant.

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